

A Guided Tour of Color Space

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This article describes the theory of color reproduction in video, and some of the engineering compromises necessary to make practical cameras and practical coding systems.

Video processing is generally concerned with color represented in three components derived from the scene, usually red, green, and blue or components computed from these. Once red, green, and blue components of a scene are obtained, color coding systems transform these components into other forms suitable for processing, recording, and transmission. Accurate color reproduction depends on knowledge of exactly how the physical spectra of the original scene are transformed into these components and exactly how the color components are transformed to physical spectra at the display.

I use the term *luminance* and the symbol Y to refer to true, CIE luminance, a linear-light quantity proportional to physical intensity. I use the term *luma* and the symbol Y' to refer to the signal that is used in video to approximate luminance. If you are unfamiliar with these terms or this usage, see page 24 of my book¹.

Trichromaticity

Color is the perceptual result of light having wavelength from 400 to 700 nm that is incident upon the retina.

The human retina has three types of color photoreceptor *cone* cells, which respond to inci-

dent radiation with different spectral response curves. Because there are exactly three types of color photoreceptors, three components are necessary and sufficient to describe a color, providing that appropriate spectral weighting functions are used: Color vision is inherently *trichromatic*.

A fourth type of photoreceptor cell, the *rod*, is also present in the retina. Rods are effective only at extremely low light intensities. Because there is only one type of rod cell, "night vision" cannot perceive color. Image reproduction takes place at light levels sufficiently high that the rod receptors play no role.

Isaac Newton said, "Indeed rays, properly expressed, are not colored." Spectral power distributions (SPDs) exist in the physical world, but color exists only in the eye and the brain.

Color specification

The Commission Internationale de L'Éclairage (CIE) has defined a system to compute a triple of numerical components that can be considered to be the mathematical coordinates of color space. Their function is analogous to coordinates on a map. Cartographers have different map projections for different functions: Some map projections preserve areas, others show latitudes and longitudes as straight lines. No single map projection fills all the needs of map users. Similarly, no single color system fills all of the needs of color users.



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¹ Charles Poynton, *A Technical Introduction to Digital Video* (New York: John Wiley & Sons, 1996)

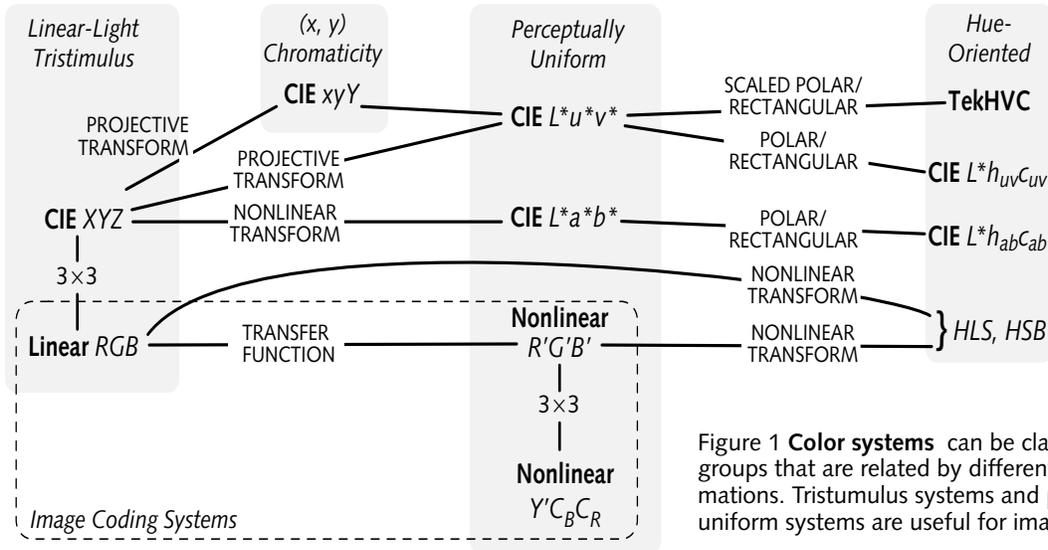


Figure 1 **Color systems** can be classified into four groups that are related by different kinds of transformations. Tristimulus systems and perceptually-uniform systems are useful for image coding.

Any system for color specification must be intimately related to the CIE specifications. The systems useful for color specification are all based on CIE XYZ. Numerical specification of hue and saturation has been standardized by the CIE, but is not useful for color specification.

A color specification system needs to be able to represent any color with high precision. Since few colors are handled at a time, a specification system can be computationally complex.

An ink color can be specified by the proportions of standard or proprietary inks that can be mixed to make the color. Ink mixture is beyond the scope of this paper.

The science of colorimetry forms the basis for describing a color as three numbers. However, classical colorimetry is intended for the specification of color, not for color image coding. Although an understanding of colorimetry is necessary to achieve good color performance in video, strict application of colorimetry is unsuitable.

Color image coding

A color image is represented as an array of pixels, where each pixel contains numerical components that define a color. Three components are necessary and sufficient for this

purpose, although in printing it is convenient to use a fourth (black) component.

In theory, the three numerical values for image coding could be provided by a color specification system. But a practical image coding system needs to be computationally efficient; cannot afford unlimited precision; need not be intimately related to the CIE system; and generally needs to cover only a reasonably wide range of colors and not all of the colors. So image coding uses different systems than color specification.

The systems useful for image coding are linear *RGB*, nonlinear *RGB*, nonlinear *CMY*, nonlinear *CMYK*, and derivatives of nonlinear *RGB* such as *Y'CbCr*. Numerical values of hue and saturation are not useful in color image coding.

If you manufacture cars, you have to match the color of paint on the door with the color of paint on the fender. A color specification system will be necessary. However, to convey a picture of the car, you need image coding. You can afford to do quite a bit of computation in the first case because you have only two colored elements, the door and the fender. In the second case, the color coding must be quite efficient because you may have a million colored elements or more.

For a highly readable short introduction to color image coding, consult DeMarsh and

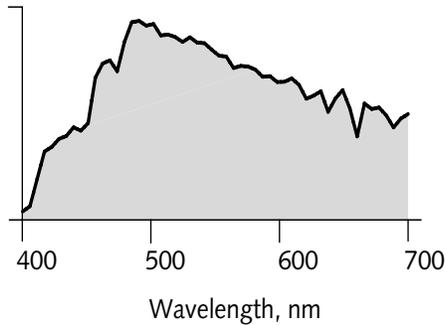
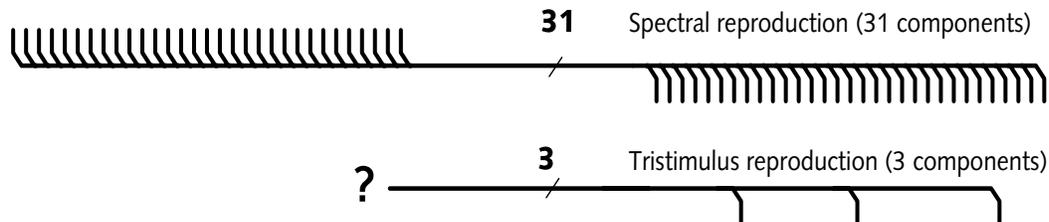


Figure 2 **Spectral and tristimulus color reproduction.** A color can be described as a spectral power distribution (SPD) of perhaps 31 numbers, each representing power over a 10 nm band. However, if appropriate spectral weighting functions are used, three values are sufficient.



Giorgianni². For a terse, complete technical treatment, read Schreiber³.

Definitions

Intensity is a measure over some interval of the electromagnetic spectrum of the flow of power that is radiated from, or incident on, a surface. Intensity is what I call a *linear-light* measure, expressed in units such as watts per square meter.

Brightness is defined by the CIE as “the attribute of a visual sensation according to which an area appears to exhibit more or less light.” Because brightness perception is very complex, the CIE defined a more tractable quantity, *luminance*, which is radiant power weighted by a spectral sensitivity function that is characteristic of vision.

Hue is the attribute of a color perception denoted by blue, green, yellow, red, and so on. Roughly speaking, if the dominant wavelength of an SPD shifts, the hue of the associated color will shift.

Saturation, or *purity*, is the degree of colorfulness, from neutral gray through pastel to satu-

rated colors. Roughly speaking, the more concentrated an SPD is at one wavelength, the more saturated the associated color. You can desaturate a color by adding light that contains the power at all wavelengths.

Spectral Power Distribution (SPD) and tristimulus

Physical power (or *radiance*) is expressed in a *spectral power distribution (SPD)*, often in 31 components each representing power in a 10 nm band from 400 to 700 nm. The SPD of the CIE standard daylight illuminant CIE D_{65} is sketched at the top of Figure 2 above.

One way to describe a color is to directly reproduce its spectral power distribution. This is sketched in the middle row of Figure 2, where 31 components are transmitted. It is reasonable to use this method to describe a single color or a few colors, but using 31 components for each pixel is an impractical way to code an image. Based on the trichromatic nature of vision, we can determine suitable spectral weighting functions to describe a color using just three components, indicated at the bottom of Figure 2.

The relationship between SPD and perceived color is the concern of the science of *colorimetry*. In 1931, the CIE adopted standard curves for a hypothetical *Standard Observer*. These curves specify how an SPD can be transformed into a triple that specifies a color.

² LeRoy E. DeMarsh and Edward J. Giorgianni, “Color Science for Imaging Systems,” in *Physics Today*, September 1989, 44-52.

³ W.F. Schreiber, *Fundamentals of Electronic Imaging Systems*, Third Edition (New York: Springer-Verlag, 1993).

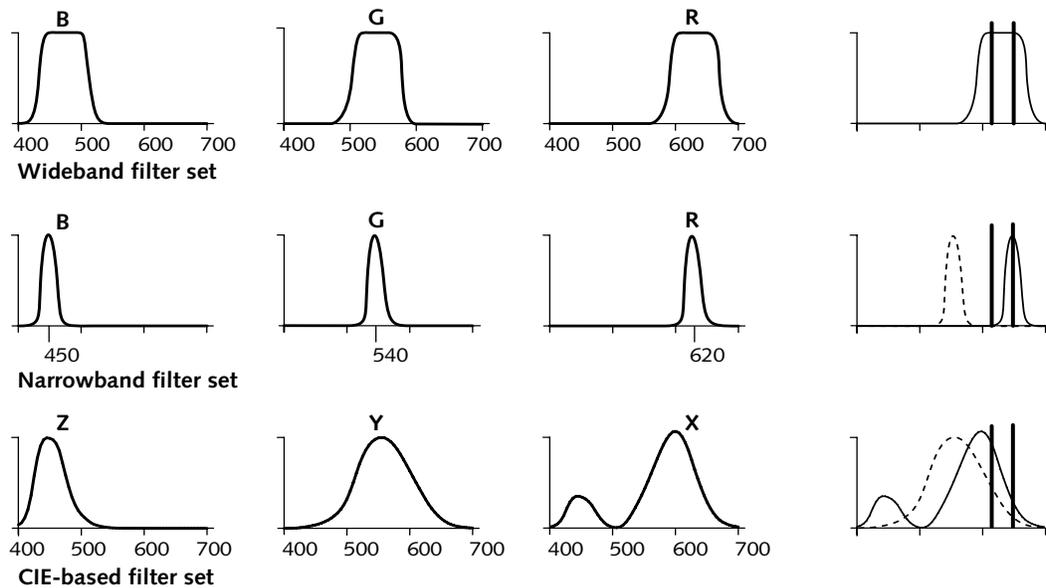


Figure 3 **Scanner spectral constraints.** This diagram shows the spectral constraints associated with scanners and cameras. The top row shows the spectral sensitivity of three wideband filters having uniform response across the shortwave, mediumwave and longwave regions of the spectrum. The problem with this approach is that two monochromatic sources seen by the eye to have different colors – in this case, saturated orange and a saturated red – cannot be distinguished by the filter set.

The middle row of the illustration shows three narrowband filters. This set solves the problem of the first set, but creates another: many monochromatic sources “fall between” the filters, and are seen by the scanner as black. To see color as the eye does, the three filter responses must be closely related to the color response of the eye.

The bottom row shows the spectral response functions of the CIE Standard Observer.

The CIE system is immediately and almost universally applicable to self-luminous sources and displays. However, the colors produced by reflective systems such as photography, printing or paint are a function not only of the colorants but also of the SPD of the ambient illumination. If your application has a strong dependence upon the spectrum of the illuminant, you may have to resort to spectral matching.

Scanner spectral constraints

The relationship between spectral distributions and the three components of a color value is conventionally explained starting from the famous color matching experiment. I will instead explain the relationship by illustrating the practical concerns of engineering the spectral filters required by a color scanner or camera, as illustrated in Figure 3 above.

The top row shows the spectral sensitivity of three wideband filters having uniform response across each of the red, green, and blue regions

of the spectrum. In a typical filter, whether for electrical signals or for optical power, it is generally desirable to have a response that is as uniform as possible across the passband, to have transition zones as narrow as possible, and to have maximum possible attenuation in the stopbands. Entire textbooks are devoted to filter design; they concentrate on optimizing filters in that way. The example on the top row of the illustration shows two monochromatic sources which are seen as saturated orange and red. Applying a “textbook” filter to that example causes these two different wavelength distributions to report the identical *RGB* triple, [1, 0, 0]. That is a serious problem, because these SPDs are seen as different colors!

At first sight it may seem that the problem with the wideband filters is insufficient wavelength discrimination. The middle row of the example addresses the apparent lack of spectral discrimination of the filters of the top row, by using three narrowband filters. This set solves one problem,

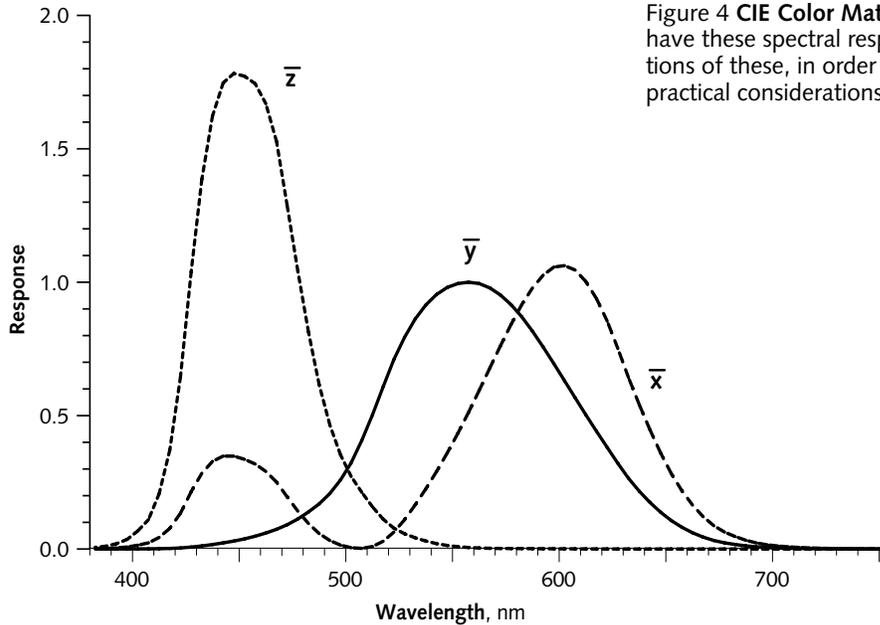


Figure 4 **CIE Color Matching Functions**. A camera must have these spectral response curves, or linear combinations of these, in order to capture all colors. However, practical considerations make this difficult.

but creates another: many monochromatic sources “fall between” the filters, and are seen by the scanner as black. In the example, the orange source reports an *RGB* triple of $[0, 0, 0]$, identical to the result of scanning black.

Although this example is contrived, the problem is not. Ultimately, the test of whether a camera or scanner is successful is whether it reports distinct *RGB* triples if and only if human vision sees different colors. To see color as the eye does, the three filter sensitivity curves must be closely related to the color response of the eye.

A famous experiment, the *color matching experiment*, was devised during the 1920s to characterize the relationship between SPD and color. The experiment measures mixtures of different spectral distributions that are required for human observers to match colors. Exploiting this indirect method, the CIE in 1931 standardized a set of spectral weighting functions that models the perception of color. These curves, defined numerically, are referred to as the \bar{x} , \bar{y} , and \bar{z} *color matching functions (CMFs)* for the CIE Standard Observer⁴. They are illustrated in the bottom row of Figure 3 opposite, and are graphed at a larger scale in Figure 4 above.

CIE XYZ tristimulus

The CIE designed their system so that one of the three tristimulus values – the *Y* value – has a spectral sensitivity that corresponds to the lightness sensitivity of human vision. The *luminance* *Y* of a source is obtained as the integral of its SPD weighted by the \bar{y} color matching function.

When luminance is augmented with two other components *X* and *Z*, computed using the \bar{x} and \bar{z} color matching functions, the resulting (X, Y, Z) components are known as *XYZ tristimulus values* (pronounced “*big-X, big-Y, big-Z*” or “*cap-X, cap-Y, cap-Z*”). These are linear-light values that embed the spectral properties of human color vision.

Tristimulus values are computed from continuous SPDs by integrating the SPD using the \bar{x} , \bar{y} , and \bar{z} color matching functions. In discrete form, tristimulus values can be computed by a matrix multiplication, as shown in Figure 5 overleaf.

⁴ The CIE system is standardized in Publication CIE No 15.2, *Colorimetry, Second Edition* (Vienna, Austria: Central Bureau of the Commission Internationale de L’Éclairage, 1986).

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.0143 & 0.0004 & 0.0679 \\ 0.0435 & 0.0012 & 0.2074 \\ 0.1344 & 0.0040 & 0.6456 \\ 0.2839 & 0.0116 & 1.3856 \\ 0.3483 & 0.0230 & 1.7471 \\ 0.3362 & 0.0380 & 1.7721 \\ 0.2908 & 0.0600 & 1.6692 \\ 0.1954 & 0.0910 & 1.2876 \\ 0.0956 & 0.1390 & 0.8130 \\ 0.0320 & 0.2080 & 0.4652 \\ 0.0049 & 0.3230 & 0.2720 \\ 0.0093 & 0.5030 & 0.1582 \\ 0.0633 & 0.7100 & 0.0782 \\ 0.1655 & 0.8620 & 0.0422 \\ 0.2904 & 0.9540 & 0.0203 \\ 0.4334 & 0.9950 & 0.0087 \\ 0.5945 & 0.9950 & 0.0039 \\ 0.7621 & 0.9520 & 0.0021 \\ 0.9163 & 0.8700 & 0.0017 \\ 1.0263 & 0.7570 & 0.0011 \\ 1.0622 & 0.6310 & 0.0008 \\ 1.0026 & 0.5030 & 0.0003 \\ 0.8544 & 0.3810 & 0.0002 \\ 0.6424 & 0.2650 & 0.0000 \\ 0.4479 & 0.1750 & 0.0000 \\ 0.2835 & 0.1070 & 0.0000 \\ 0.1649 & 0.0610 & 0.0000 \\ 0.0874 & 0.0320 & 0.0000 \\ 0.0468 & 0.0170 & 0.0000 \\ 0.0227 & 0.0082 & 0.0000 \\ 0.0114 & 0.0041 & 0.0000 \end{bmatrix}^T \cdot \begin{bmatrix} 82.75 \\ 91.49 \\ 93.43 \\ 86.68 \\ 104.86 \\ 117.01 \\ 117.81 \\ 114.86 \\ 115.92 \\ 108.81 \\ 109.35 \\ 107.80 \\ 104.79 \\ 107.69 \\ 104.41 \\ 104.05 \\ 100.00 \\ 96.33 \\ 95.79 \\ 88.69 \\ 90.01 \\ 89.60 \\ 87.70 \\ 83.29 \\ 83.70 \\ 80.03 \\ 80.21 \\ 82.28 \\ 78.28 \\ 69.72 \\ 71.61 \end{bmatrix} \begin{matrix} 400 \text{ nm} \\ \\ \\ \\ \\ 450 \text{ nm} \\ \\ \\ \\ \\ 500 \text{ nm} \\ \\ \\ \\ \\ 550 \text{ nm} \\ \\ \\ \\ \\ 600 \text{ nm} \\ \\ \\ \\ \\ 650 \text{ nm} \\ \\ \\ \\ \\ 700 \text{ nm} \end{matrix}$$

Figure 5 **Calculation of tristimulus values by matrix multiplication.** The column vector at the right is a discrete version of CIE Illuminant D_{65} . The 31-by-3 matrix is a discrete version of the set of CIE x , y and z color matching functions, here represented in 31 wavelengths at 10 nm intervals across the spectrum. The result of performing the matrix multiplication is a set of XYZ tristimulus values.

Human color vision follows a principle of superposition, first elaborated by Grassman and now known as Grassman's Law: The tristimulus values that result after summing a set of SPDs is identical to the sum of the tristimulus values of each of the SPDs. Due to this linearity of additive color mixture, any set of three components that is a nontrivial linear combination of X , Y , and Z is also a set of tristimulus values.

The CIE system is based on the description of color as a luminance component Y , as described above, and two additional components X and Z . The spectral weighting curves of X and Z have been standardized by the CIE based on statistics from experiments involving human observers. XYZ tristimulus values can describe any color. (RGB tristimulus values will be described later.)

The magnitudes of the XYZ components are proportional to physical power, but their spectral composition corresponds to the color matching characteristics of human vision.

CIE x , y chromaticity

It is convenient, for both conceptual understanding and computation, to have a representation of "pure" color in the absence of luminance. The CIE standardized a procedure for normalizing XYZ tristimulus values to obtain two chromaticity values x and y (pronounced "little- x , little- y "). The relationships are computed by this projective transformation:

$$\text{Eq 1} \quad x = \frac{X}{X+Y+Z} \quad y = \frac{Y}{X+Y+Z}$$

A color plots as a point in an (x, y) chromaticity diagram, illustrated in Figure 6 opposite. When a narrowband SPD comprising power at just one wavelength is swept across the range 400 to 700 nm, it traces a shark-fin shaped spectral locus in (x, y) coordinates. The sensation of purple cannot be produced by a single wavelength: To produce purple requires a mixture of shortwave and longwave light. The line of purples joins extreme blue to extreme red. The chromaticity coordinates of real (physical) SPDs are bounded by the line of purples and the spectral locus: All colors are contained in this region of the chromaticity diagram.

It is common to specify a color by its chromaticity and luminance, in the form of an xyY triple. (A third chromaticity value, z , can be computed similarly. However, z is redundant if x and y are known, due to the identity $x + y + z = 1$.)

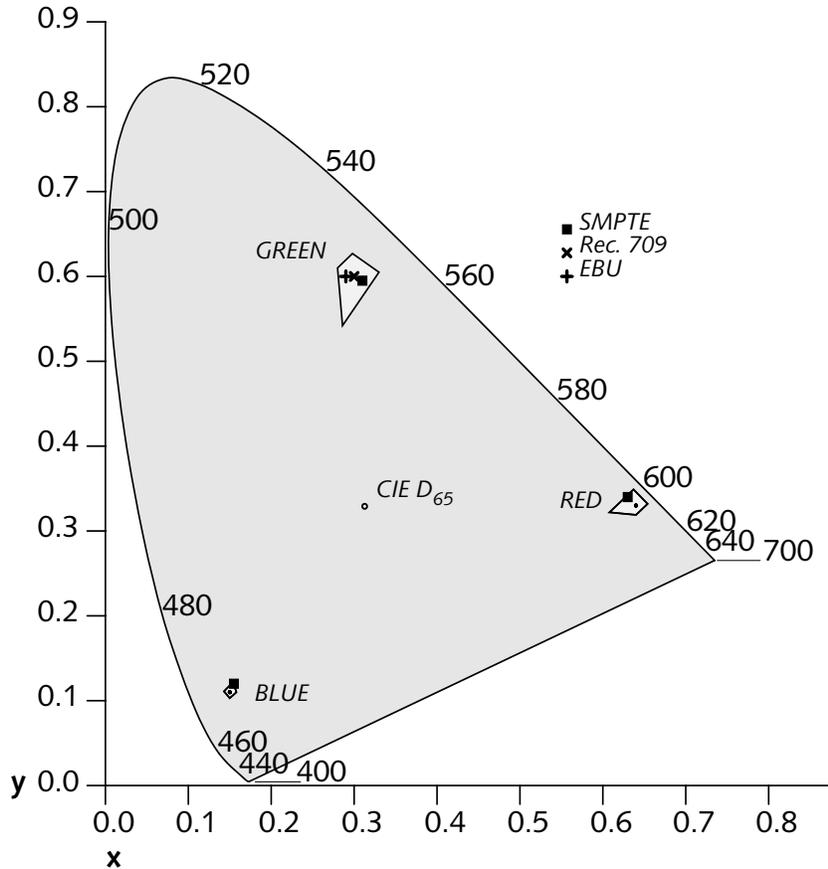


Figure 6 **The (x, y) chromaticity diagram.** The *spectral locus*, an inverted U-shaped path, is swept out as a monochromatic source runs from 400 to 700 nm. The locus of all colors is closed by the *line of purples*, which traces SPDs that combine longwave and shortwave power but have no mediumwave contribution.

On this chromaticity diagram, I plot the chromaticity of the D_{65} white point, and the chromaticities of the red, green, and blue primaries of contemporary SMPTE, EBU, and ITU-R standards for video primaries. See *A Technical Introduction to Digital Video* for details.

To recover X and Z tristimulus values from chromaticities and luminance, use the inverse of Eq 1:

$$\text{Eq 2} \quad X = \frac{x}{y} Y \quad Z = \frac{1-x-y}{y} Y$$

The projective transformation used to compute x and y is such that any linear combination of two spectra, or two tristimulus values, plots on a straight line in the (x, y) plane. However, the transformation does not preserve distances, so chromaticity values do not combine linearly.

There is no unique physical or perceptual definition of white. An SPD that is considered white will have CIE (x, y) coordinates roughly in the central area of the chromaticity diagram, in the region of $(\frac{1}{3}, \frac{1}{3})$.

Additive mixture (RGB)

The simplest way to reproduce a wide range of colors is to mix light from three lights of different colors, usually red, green, and blue. Figure 7

overleaf illustrates additive reproduction. In physical terms, the spectra from each of the lights add together wavelength by wavelength to form the spectrum of the mixture.

As a consequence of the principle of superposition, the color of an additive mixture is a strict function of the colors of the primaries and the fraction of each primary that is mixed.

Additive reproduction is employed directly in a video projector, where the spectra from a red beam, a green beam, and a blue beam are physically summed at the surface of the projection screen.

Additive reproduction is also employed in a direct-view color CRT, but through slightly indirect means. The screen of a CRT comprises small dots that produce red, green, and blue light. When the screen is viewed from a sufficient distance, the spectra of these dots add at the observer's retina.

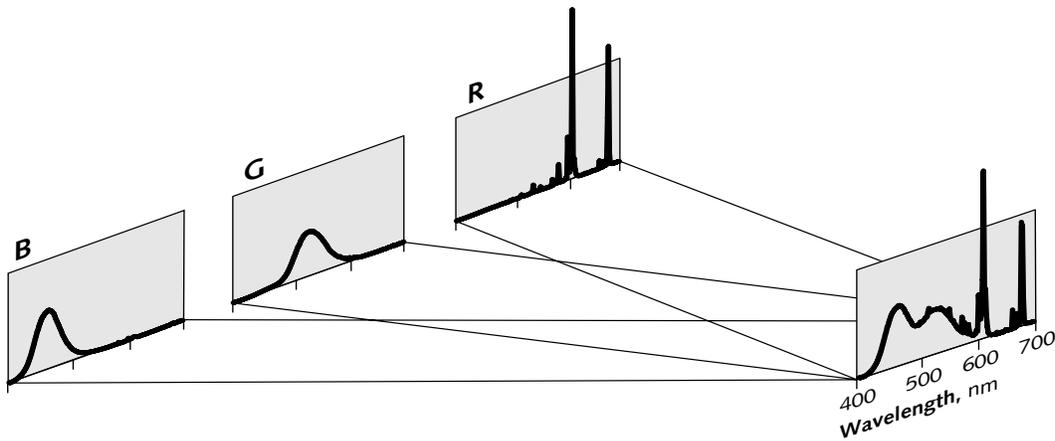


Figure 7 **Additive mixture.** This diagram illustrates the physical process underlying additive color mixture, as is used in color television. Each colorant has an independent, direct path to the image. The spectral power of the image is, at each wavelength, the sum of the spectra of

the colorants. The colors (chromaticities) of the mixtures are completely determined by the colors of the primaries, and analysis and prediction of mixtures is reasonably simple. The spectral curves shown here are those of an actual Sony Trinitron™ monitor.

White

In additive image reproduction, the *white point* is the chromaticity of the color reproduced by equal red, green, and blue components. White point is a function of the ratio (or *balance*) of power among the primaries.

It is often convenient for purposes of calculation to define white as a uniform SPD. This white reference is known as the *equal-power illuminant*, or *CIE Illuminant E*.

A more realistic reference that approximates daylight has been specified numerically by the CIE as Illuminant D_{65} . You should use this unless you have a good reason to use something else. The print industry commonly uses D_{50} , and photography commonly uses D_{55} . These represent compromises between the conditions of indoor (tungsten) and daylight viewing.

Human vision adapts to white in the viewing environment. An image viewed in isolation – such as a slide projected in a dark room – creates its own white reference, and a viewer will be quite tolerant of errors in the white point. However, if the same image is viewed in the presence of an external white reference or a second image, then differences in white point can be objectionable.

Planck determined that the SPD radiated from a hot object – a *black body radiator* – is a function of the temperature to which the object is heated. A typical source of illumination has a heated object at its core, so it is often useful to characterize an illuminant by specifying the temperature (in units of kelvin, K) of a black body radiator that appears to have the same color.

Although an illuminant can be specified informally by its color temperature, a more complete specification is provided by the chromaticity coordinates of the SPD of the source. Figure 8 shows the SPDs of several standard illuminants.

Camera white reference

There is an implicit assumption in television that the camera operates as if the scene were illuminated by a source having the chromaticity of CIE D_{65} . In practice, the scene illumination is often deficient in the shortwave (blue) region of the spectrum. Television studio lighting is often accomplished by tungsten lamps, which are very yellow. This situation is accommodated by *white balancing*, that is, by adjusting the gain of the red, green, and blue components of the scene so that a white object reports the values that would be reported if the scene were illuminated by D_{65} .

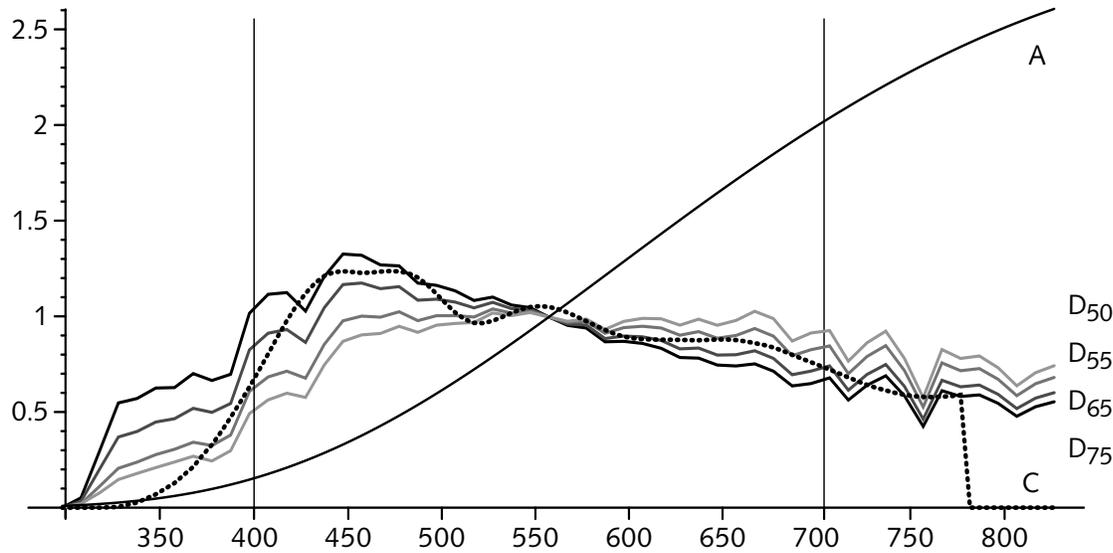


Figure 8 CIE illuminants are graphed here. Illuminant A is now obsolete; it is representative of tungsten illumination. Illuminant C was an early standard for daylight; it

too is obsolete. The family of D Illuminants represent daylight at several color temperatures.

Monitor white reference

In an additive mixture the illumination of the reproduced image is generated entirely by the display device. In particular, reproduced white is determined by the characteristics of the display, and is not dependent on the environment in which the display is viewed. In a completely dark viewing environment such as a cinema theater, this is desirable. However, in an environment where the viewer's field of view encompasses objects other than the display, the viewer's notion of "white" is likely to be influenced or even dominated by what he perceives as "white" in the ambient. To avoid subjective mismatches, the chromaticity of white reproduced by the display and the chromaticity of white in the ambient should be reasonably close.

Modern blue CRT phosphors are more efficient with respect to human vision than red or green. In a quest for brightness at the expense of color accuracy, it is common for a computer display to have excessive blue content, about twice as blue as daylight, with white at about 9300 K. This situation can be corrected by calibrating the monitor to a white reference with a lower color temperature.

Characterization of RGB primaries

Additive reproduction is based on physical devices that produce all-positive SPDs for each primary. Physically and mathematically, the spectra add. The largest range of colors will be produced with primaries that appear red, green, and blue. Human color vision obeys the principle of superposition, so the color produced by any additive mixture of three primary spectra can be predicted by adding the corresponding fractions of the XYZ components of the primaries: the colors that can be mixed from a particular set of RGB primaries are completely determined by the colors of the primaries by themselves. Subtractive reproduction is much more complicated: the colors of mixtures are determined by the primaries and by the colors of their combinations.

An additive RGB system is specified by the chromaticities of its primaries and its white point. The extent (*gamut*) of the colors that can be mixed from a given set of RGB primaries is given in the (x, y) chromaticity diagram by a triangle whose vertices are the chromaticities of the primaries.

In computing, there are no standard primaries and there is no standard white point. If you have an RGB image but have no information about its

chromaticities, you cannot accurately determine the colors represented by the image data.

The NTSC in 1953 specified a set of primaries that were representative of phosphors used in color CRTs of that era. However, phosphors changed over the years, primarily in response to market pressures for brighter receivers, and by the time of the first videotape recorder the primaries in use were quite different than those “on the books.” So although you may see the NTSC primary chromaticities documented, they are of no practical use today.

Contemporary studio monitors have slightly different standards in North America, Europe, and Japan. However, international agreement⁵ has been obtained on primaries for high-definition television (HDTV), and these primaries are closely representative of contemporary monitors in studio video, computing, and computer graphics. The standard is formally denoted *ITU-R Recommendation BT. 709* (formerly CCIR Rec. 709). I’ll call it *Rec. 709*. The Rec. 709 primaries and its D_{65} white point are these:

Table 1 Rec. 709 primaries and white reference

	<i>R</i>	<i>G</i>	<i>B</i>	white
<i>x</i>	0.640	0.300	0.150	0.3127
<i>y</i>	0.330	0.600	0.060	0.3290
<i>z</i>	0.030	0.100	0.790	0.3582

For a discussion of nonlinear *RGB* in computer graphics, read Lindbloom’s SIGGRAPH paper⁶. For technical details on monitor calibration, consult Cowan⁷.

Video standards specify abstract *R’G’B’* systems that are closely matched to the characteristics of

real monitors. Physical devices that produce additive color involve tolerances and uncertainties, but if you have a monitor that conforms to Rec. 709 within some tolerance, you can consider the monitor to be device-independent.

The importance Rec. 709 as an interchange standard in studio video, broadcast television, and high definition television, and the perceptual basis of the standard, assures that its parameters will be used even by devices such as flat-panel displays that do not have the same physics as CRTs.

Non-realizable spectra

In the \bar{x} , \bar{y} , and \bar{z} functions, the contribution of each weighting function to each tristimulus value is positive at every wavelength. Light cannot convey negative power; if a color matching function had a negative excursion, its filter could not be physically realized.

If you examine the \bar{x} , \bar{y} , and \bar{z} functions, you will see that – except for the trivial case of an identically zero spectrum – it is not possible to have a physical spectrum that produces zero luminance: Every wavelength has some positive contribution to *Y*. There is no way to generate nonzero *X* or *Z* values without also generating nonzero *Y*. Thus, a “pure” *X* tristimulus triple [1, 0, 0] can never be obtained from a physical (all-positive) spectral distribution. However, setting aside for the moment the physical constraint, a pure *X* tristimulus value would correspond mathematically to a spectrum that has negative power at some wavelengths, in order to generate unity *X* and zero luminance *Y*.

The SPDs corresponding to the CIE \bar{x} , \bar{y} , and \bar{z} color matching functions correspond to physically realizable spectra, SPDs having these shapes could in theory be used as the primaries in a display device such as a CRT. However, using *XYZ* tristimulus values to drive a display having \bar{x} , \bar{y} , and \bar{z} primaries would not correctly reproduce color. This is fundamentally because the analysis of the source SPD into *X*, *Y*, and *Z* components proceeds independently during scanning, but in an additive display device, the SPDs of the primaries interact when they

⁵ *Basic Parameter Values for the HDTV Standard for the Studio and for International Programme Exchange* (Geneva, Switzerland: ITU, 1990).

⁶ Bruce Lindbloom, “Accurate Color Reproduction for Computer Graphics Applications,” in *Computer Graphics*, Vol. 23, No. 3 (July 1989), 117-126.

⁷ William B. Cowan, “An Inexpensive Scheme for Calibration of a Colour Monitor in terms of CIE Standard Coordinates,” in *Computer Graphics*, Vol. 17, No. 3 (July 1983), 315-321.

combine at the display. If power from the source could be “steered” into three components at the source – if power contributing to one component could be subtracted from the other two – then the component values at the scanner could be directly used at the display.

Unfortunately this “steering” operation involves subtraction, which in the case of the \bar{x} , \bar{y} , and \bar{z} functions leads to spectral sensitivity functions that are negative at some wavelengths, and therefore not realizable in practice.

Alternatively, the display could be arranged so that the X tristimulus component produced a contribution having a spectral power distribution $\bar{x} - \bar{y} - \bar{z}$ (and similarly for Y and Z). Unfortunately this proposal suffers from nonrealizable primaries at the display. So we have a dilemma: to utilize XYZ tristimulus components directly to achieve color reproduction, we must have either a nonrealizable scanner or a nonrealizable display!

The solution to this dilemma involves transforming the tristimulus components into a mathematical domain where physical realizability is not a constraint. The transformation depends on the fact that the principle of superposition applies to human color vision. We process the XYZ components through a 3×3 linear (matrix) multiplication, to produce a new set of components appropriate for physical RGB display primaries.

Transformations between CIE XYZ and RGB

RGB values in a particular set of primaries can be transformed to and from CIE XYZ by a 3×3 matrix transform. These transforms involve *tristimulus values*, that is, sets of three linear-light components that conform to the CIE color matching functions. SMPTE has published details on how to compute these transformations⁸. CIE XYZ is a special case of tristimulus values. In XYZ, any color is represented by a positive set of values.

To transform from CIE XYZ into Rec. 709 RGB (with its D_{65} white point), use this transform:

$$\begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix} = \begin{bmatrix} 3.240479 & -1.537150 & -0.498535 \\ -0.969256 & 1.875992 & 0.041556 \\ 0.055648 & -0.204043 & 1.057311 \end{bmatrix} \bullet \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

This matrix has some negative coefficients: XYZ colors that are *out of gamut* for a particular RGB transform to RGB where one or more RGB components is negative or greater than unity.

Here’s the inverse transform. Because white is normalized to unity, the middle row sums to unity:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.412453 & 0.357580 & 0.180423 \\ 0.212671 & 0.715160 & 0.072169 \\ 0.019334 & 0.119193 & 0.950227 \end{bmatrix} \bullet \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix}$$

To recover primary chromaticities from such a matrix, compute little x and y for each RGB column vector. To recover the white point, transform RGB = [1, 1, 1] to XYZ, then compute x and y.

Transforms among RGB systems

RGB tristimulus values in a system employing one set of primaries can be transformed into another set by a 3×3 matrix multiplication. Generally these matrices are normalized for a white point luminance of unity. For details, see *Television Engineering Handbook*⁹.

As an example, here is the transform from SMPTE 240M (or SMPTE RP-145) RGB to Rec. 709:

$$\begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix} = \begin{bmatrix} 0.939555 & 0.050173 & 0.010272 \\ 0.017775 & 0.965795 & 0.016430 \\ -0.001622 & -0.004371 & 1.005993 \end{bmatrix} \bullet \begin{bmatrix} R_{145} \\ G_{145} \\ B_{145} \end{bmatrix}$$

This matrix is very close to the identity matrix. In a case like this, if the transform is computed in the nonlinear (gamma-corrected) $R'G'B'$ domain the resulting errors will be insignificant.

⁸ SMPTE RP 177-1993, *Derivation of Basic Television Color Equations*.

⁹ *Television Engineering Handbook, Featuring HDTV Systems, Revised Edition* by K. Blair Benson, revised by Jerry C. Whitaker (New York: McGraw-Hill, 1992). This supersedes the Second Edition.

Here's another example. To transform EBU 3213 RGB to Rec. 709:

$$\begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix} = \begin{bmatrix} 1.044036 & -0.044036 & 0. \\ 0. & 1. & 0. \\ 0. & 0.011797 & 0.988203 \end{bmatrix} \bullet \begin{bmatrix} R_{EBU} \\ G_{EBU} \\ B_{EBU} \end{bmatrix}$$

Transforming among RGB systems may lead to an *out of gamut* result where one or more RGB components is negative or greater than unity.

Scanning colored media

The previous section discusses the analysis filters necessary to obtain three tristimulus values from incident light from a colored object. However, if the incident light has itself been formed by an imaging system based on three components, the spectral composition of light from the original scene has already been analyzed. To perform a spectral analysis based on the CIE color matching functions is unnecessary at best, and is likely to introduce degraded color information. A much more direct, practical, and accurate approach is to perform an analysis using filters tuned to the spectrum of the colorants of the reproduced image, and thereby extract the three color "records" present in the reproduction being scanned.

In practical terms, if an image to be scanned has already been recorded on color film, or printed by color photography or offset printing, then better results will be obtained by using three narrowband filters designed to extract the color components that have already been recorded on the media. A *densitometer* is an instrument that performs this task (but reporting logarithmic *density* values instead of linear-light tristimulus values). Several spectral sensitivity functions have been standardized for densitometers for use in different applications.

Practical camera spectral response

The previous section explained how the CIE \bar{x} , \bar{y} , and \bar{z} functions are representative of the spectral response of the three color photoreceptor cells of human vision, and explained how XYZ tristimulus components from such a scanner are transformed for use by a display device having realizable primaries. Although an ideal scanner

or a camera would use these spectral functions, many practical difficulties preclude their use in real cameras and scanners.

The CIE \bar{x} , \bar{y} , and \bar{z} functions capture all colors. However, the range of colors commonly encountered in the natural or man-made world is a fraction of total color space. Although it is useful for an instrument such as a colorimeter to measure all colors, in an imaging system we are generally concerned with the subset of colors that occur frequently. If a scanner is designed to capture all colors, its complexity is necessarily higher and its performance is necessarily worse than a camera designed to capture a smaller range of colors.

Furthermore, in a system that reproduces images, such as television, any economical display device will be limited in the range of colors that it can reproduce. There are usually good engineering reasons to capture a range of colors that is not excessively larger than the range of colors that can be reproduced. For much of the history of color television, cameras were designed to incorporate assumptions about the color reproduction capabilities of color CRTs. However, nowadays, video production equipment is being used to originate images for a wider range of applications than just television broadcast. The desire to make video cameras suitable for originating images for these wider applications has led to cameras with increased color range.

"Subtractive" mixture (CMY)

In contrast to additive mixture described earlier, another way to produce a range of color mixtures is to selectively remove portions of the spectrum from a relatively broadband illuminant. This is illustrated in Figure 9 opposite.

The illuminant at the left of the sketch produces light over most or all of the visible spectrum, and each successive filter transmits some portion of the band and attenuates other portions. The overall attenuation of each filter is controlled: In the case of color photography, this is accomplished by varying the thickness of each dye

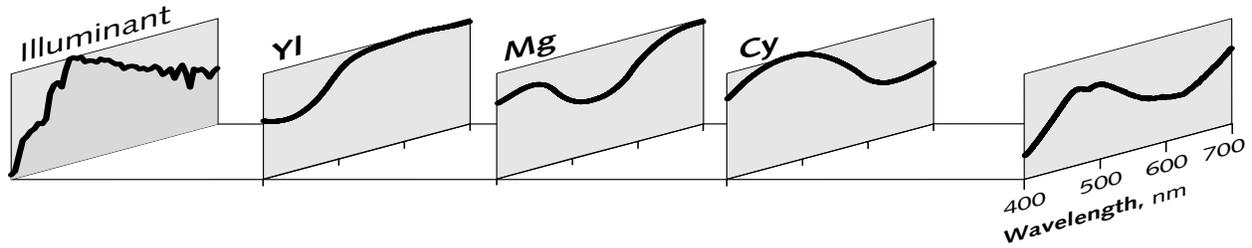


Figure 9 **Subtractive mixture** as employed in color photography and color offset printing, is illustrated in this diagram. The colorants act in succession to remove spectral power from the illuminant. In physical terms, the spectral power of the mixture is, at each wavelength, the product of the spectrum of the illuminant and the transmission of the colorants: the mixture could be called multiplicative. If the amount of each colorant is represented in the form of *density* – the negative of the base-10 logarithm of transmission – then color mixtures

can be determined by subtraction.

Color mixtures in subtractive systems are complex because the colorants have absorption not only in the intended region of the spectrum but also unwanted absorptions in other regions. The unwanted absorptions cause interaction among the channels, and therefore nonlinearity in the mixtures. The illuminant shown here is CIE D_{65} ; the colorant absorption curves are those of the dyes of a commercial color photographic print film.

layer that functions as a filter, the shape of each curve remaining constant.

In physical terms, the spectrum of the mixture is the wavelength by wavelength product of the spectrum of the illuminant and the spectral transmission curves of each of the colorants. The spectral transmission curves of the colorants multiply, so this method of color reproduction should really be called “multiplicative.” Photographers and printers have for decades measured transmission in base-10 logarithmic *density* units, where transmission of unity corresponds to a density of 0, transmission of 0.1 corresponds to a density of 1, transmission of 0.01 corresponds to a density of 2, and so on. When a printer or photographer computes the effect of filters in tandem, he subtracts density values instead of multiplying transmission values, so he calls the system *subtractive*.

To achieve a wide range of colors in a subtractive system requires filters that appear colored cyan, yellow, and magenta (*CMY*). Figure 9 illustrates the situation of a transmissive display such as in the projection of color photographic film.

RGB information can be used as the basis for subtractive image reproduction. If the color to be reproduced has a blue component of zero, then

the yellow filter must attenuate the shortwave components of the spectrum as much as possible. As the amount of blue to be reproduced increases, the attenuation of the yellow filter should decrease. This reasoning leads to the “one-minus-*RGB*” relationships:

$$\begin{aligned} Cy &= 1 - R \\ Mg &= 1 - G \\ YI &= 1 - B \end{aligned}$$

Cyan in tandem with magenta produces blue, cyan with yellow produces green, and magenta with yellow produces red. Here is a memory aid:

R	G	B	R	G	B
Cy	Mg	YI	Cy	Mg	YI

Additive primaries are at the top, subtractive at the bottom. On the left, magenta and yellow filters combine subtractively to produce red. On the right, red and green sources combine additively to produce yellow.

Subtractive details

Although the “one-minus-*RGB*” relationships are a good starting point, they are not useful in practice. One problem is that the relationships assume *RGB* to represent linear-light tristimulus values and *CMY* to represent linear-light trans-

mission values. In computing and in offset printing, neither the *RGB* nor the *CMY* quantities are directly proportional to intensity. Another problem is that the “one-minus-*RGB*” relationships do not incorporate any about information about tristimulus values of the assumed *RGB* primaries or the assumed *CMY* colorants! Obviously the spectra of the colorants has some influence over reproduced color, but the model embeds no information about the spectra!

The most serious practical problem with “subtractive” color mixture is that any overlap among the absorption spectra of the colorants results in “unwanted absorption” in the mixture. For example, the magenta dye is intended to absorb only medium wavelengths, roughly between 500 and 600 nm. However, for the photographic film dyes shown in Figure 9, the magenta dye also has substantial absorption in the shortwave region of the spectrum. Consequently, the amount of shortwave light that contributes to any mixture is not only a function of the amount of attenuation introduced by the yellow filter, but also the incidental attenuation introduced by the magenta filter. There is no simple way to compensate these interactions, because the multiplication of spectra is not mathematically a linear operation.

In practice, the conversion from *RGB* to *CMY* is accomplished using either fairly complicated polynomial arithmetic, or by three-dimensional interpolation of lookup tables.

In a subtractive mixture, reproduced white is determined by characteristics of the colorants and by the spectrum of the illuminant used at the display. In a reproduction such as a color photograph that is illuminated by the ambient light in the viewer’s environment, mismatch between the white reference in the scene and the white reference in the viewing environment is eliminated. In a subtractive reproduction using a device that generates its own illumination, the subjective effect of white mismatch may need to be considered.

Image synthesis

Once light is on its way to the eye, any tristimulus-based system can accurately represent color. However, the interaction of light and objects involves SPDs and spectral absorption curves, not tristimulus values. In synthetic computer graphics, the calculations are actually simulating sampled SPDs, even if only three components are used. Details concerning the resultant errors are found in Hall¹⁰.

¹⁰ Roy Hall, *Illumination and Color in Computer Generated Imagery* (New York: Springer-Verlag, 1989)