

The magnitude of nonconstant luminance errors

Charles Poynton,
*A Technical Introduction
to Digital Video*. New York:
John Wiley & Sons, 1996.

This note outlines the magnitude of the errors introduced by the failure of contemporary video systems to adhere to the *Principle of constant luminance*. I assume that you are familiar with chapters 5 through 9 of my book, and that you are aware of the ambiguity and confusion in television engineering concerning the term *luminance*.

I express signal processing operations in Mathematica-like pseudocode. The midline dot symbol (\cdot) represents the dot product operation.

To compute true, CIE luminance from tristimulus (linear light) *RGB* components according to Rec. 709 requires weighting the *RGB* components with these luminance coefficients:

$$\text{Eq 1} \quad Luma_{709} = \{0.2125, 0.7154, 0.0721\}$$

Consider a fully-saturated magenta pixel, represented in Rec. 709 *RGB* primaries. Its *RGB* (tristimulus, linear light) components are {1, 0, 1}. Its true, CIE luminance value is this:

$$\text{Eq 2} \quad Luma_{709} \cdot \{1, 0, 1\}$$
$$0.2846$$

In video, we do not compute a true, CIE luminance signal according to the principles of color science. Instead, we gamma-correct the *RGB* components, then sum them to form a nonlinear component that I denote *luma*. If you approximate gamma correction as a square root, the encoded *luma* value is this:

$$\text{Eq 3} \quad Luma_{709} \cdot \{\text{Sqrt}[1], \text{Sqrt}[0], \text{Sqrt}[1]\}$$
$$0.533479$$

At the color difference decoder, we reconstruct gamma-corrected *R'G'B'* components {1, 0, 1}; these are presented to the display. The display imposes approximately a 2.5-power function. For the purposes of this note, we can approximate the power function as a square. The

power function has no effect on the components of magenta in this case, since both 0 and 1 are unchanged by the power function.

Luminance will be perceived from the display in accordance with the principles of color science:

$$\text{Eq 4} \quad Luma_{709} \cdot \{1, 0, 1\}$$

$$0.2846$$

No error is introduced at DC. The problem arises from the fact that high-frequency information is filtered out of the color difference components. In this case, the $Y'P_BP_R$ components are these:

$$\text{Eq 5} \quad encode_{709} = Y'P_BP_R_RGB[Luma_{709}, \{0, 0, 0\}, \{1, .5, .5\}]$$

$$\text{Eq 6} \quad encode_{709} \cdot \{1, 0, 1\}$$

$$0.2846$$

$$0.385494$$

$$0.454222$$

In the case of high-frequency chroma, the P_B and P_R components are attenuated to zero. When decoded, the reconstructed $R'G'B'$ components are these:

$$\text{Eq 7} \quad \text{Inverse}[encode_{709}] \cdot \{0.2846, 0, 0\}$$

$$0.2846$$

$$0.2846$$

$$0.2846$$

These nonlinear quantities are squared by the display:

$$\text{Eq 8} \quad (\text{Inverse}[encode_{709}] \cdot \{0.2846, 0, 0\})^2$$

$$0.080997$$

$$0.080997$$

$$0.080997$$

This combination of RGB tristimulus values is perceived according to the principles of color science:

$$\text{Eq 9} \quad Luma_{709} \cdot (\text{Inverse}[encode_{709}] \cdot \{0.2846, 0, 0\})^2$$

$$0.080997$$

You can see the magnitude of the problem: a scene element with a true luminance of 0.413 is reproduced with a luminance of less than 25 percent of that value.

Encoding and decoding according to the luma coefficients of Rec. 601 instead of Rec. 709 reconstructs luminance as:

$$\text{Eq 10} \quad \text{encode}_{601} = Y'P_B P_R \text{RGB}[Luma_{601}, \{0, 0, 0\}, \{1, .5, .5\}]$$

$$\text{Eq 11} \quad Luma_{709} \cdot (\text{Inverse}[\text{encode}_{601}] \cdot \{0.413, 0, 0\})^2 \\ 0.170569$$

This error is an intrinsic part of video. Nothing short of rearranging the block diagram of television will eliminate it. Changing the luma coefficients changes the magnitude of the error, but that does not make it go away. The good news is that we have about four decades of experience with these errors, and they are not that severe.

The current approach in HDTV standardization is to adopt new luma coefficients, different from those of conventional video.

I argue that while using the Rec. 709 coefficients might be more theoretically appropriate than using Rec. 601 coefficients, we would still be using the coefficients in the wrong equation! The intrinsic errors introduced into high-frequency chroma overwhelm any potential theoretical advantage of optimizing the coefficients. No practical benefit is obtained by changing them.

On the other hand, making the change introduces complexity into upconversions and downconversions between conventional (Rec. 601) video and HDTV. If the increased complexity is avoided by omitting to perform luma/color difference processing in upconverters and downconverters, then large color errors will be introduced, not only into the high frequency components of the signal, but much more significantly, into DC and low frequencies. Errors in low frequencies will be quite visible, and in my opinion, objectionable. The companion note *Transforms between luma coefficient sets* provides the conversion matrices, and indicates the magnitude of the errors that will be introduced if the matrices are not used.